Economic capital – how much do you really need?

This month sees the start of Charles Smithson's third series of Class Notes, a series that will run in alternate issues of *Risk* through to the end of 2004. Class Notes is an educational series, designed to pull together the threads of recent developments and thinking about key issues in risk management and derivatives dealing.

Many of *Risk*'s readers were probably not in the market – or even in college – when the first series of Class Notes started back in December 1992. In his inaugural article for that first series, entitled *Something or Nothing*, Smithson (pictured here) set out to demonstrate how hedging could add value to a firm.

Times change, but this new series of Class Notes is no less concerned with the fundamental issues challenging professionals.

This latest instalment deals with perhaps the most fundamental of risk fundamentals – economic capital, and how it is measured and attributed. In the January issue of *Risk*, the Class Notes topic will be loan valuation.

Economic capital is becoming the language of risk. While market, credit and operational risk have different determinants and use different methodologies, the levels of risk can all be summarised in a common dimension – the amount of economic capital needed to support the risk.

Economic capital is determined by the riskiness of the firm's assets (business units and activities) – more risk requires more capital. To decide how much economic capital to hold, most financial firms are looking at the "target insolvency rate" selected by senior management and the board -- the lower the insolvency rate the firm targets, the more economic capital the firm will need to hold.[1] Equity capital is the most likely way that a firm will satisfy the economic capital requirement, but it is not the only way. For a given slate of assets, insurance and guarantees reduce the amount of economic capital required, by transferring risk from the firm to a third party.[2] And structures such as a credit line that can be accessed in all conditions can be substitutes for equity.

It appears that Bankers Trust was the first to embrace the economic capital concept, as it developed risk-adjusted return on (economic) capital (Raroc). To implement a risk-adjusted performance measure – Raroc or shareholder-value-added (SVA) – the firm must be able to accomplish one difficult task (measure the amount of economic capital needed to support the entire firm) and one extremely difficult task (attribute economic capital to individual business units and transactions). It is with these tasks that this column will deal.

Task one: measure economic capital at the firm level

Merton & Perold (1993) defined what we call 'economic capital' as "the smallest amount that can be invested to insure the value of the firm's net assets against a loss in value relative to the risk-free investment of those net assets".[3] So far, we have seen three methodologies suggested for measuring economic capital at the firms level.

☐ **The 'top-down' approach.** Matten (2000) demonstrated that total economic capital could be estimated using an estimate of the volatility of the firm's earnings. Suppose a company was able to calculate the monthly mean and standard deviation of its earnings as $100 million and $30 million respectively. Suppose further that, for the firm to be a going concern, monthly pre-tax profit must be at least $40 million, that is, 'earnings at risk' equal $40 million.[4] The amount of capital necessary to support this firm is the amount of capital that would have to be invested at the risk-free rate to generate a monthly flow of $40 million. That is:

$$\text{Economic capital} = \frac{\text{Earnings at risk}}{r}$$

where r is the risk-free interest rate. If we assume the 30-year US Treasury rate is 5%, the monthly rate r turns out to be 0.41%, and the amount of capital that will generate a monthly flow of $40 million is $40 million divided by 0.0041, or $9.6 billion.

The problem with this approach is that it requires the firm to have enough historical data to obtain reliable estimates of the mean and standard deviation of its earnings, and few companies have enough high-frequency data to yield reliable estimates of earnings volatility.

☐ **The 'bottom-up' approach.** It appears that most financial institutions are using a bottom-up approach to estimate total economic capital for the firm. The 'bottom-up' designation derives from the fact that individual transactions are modelled and then aggregated. The financial institutions that use this approach obtain

[1] More often, people talk about the 'confidence level' used in determining economic capital. The confidence level is simply one minus the target insolvency rate

[2] Indeed, in their 1993 paper, Merton & Perold noted that economic capital can be viewed as providing a kind of 'asset insurance' against the possibility of lower-than-expected operating results

[3] Merton & Perold called this 'risk capital'. Coming at it from a slightly different angle, Shimpi (2002) defined 'risk capital' as the amount of equity capital in excess of the firm's 'operational capital' that is required to limit the probability of financial distress to a level considered acceptable by top management. Shimpi would then call 'economic capital' the sum of 'risk capital' and 'operational capital'. (Shimpi argues that firms may also choose to hold 'signalling capital' to reassure investors, stock analysts, customers, suppliers, regulators and rating agencies that the firm is sound as the managers know it to be)

[4] Note that this earnings-at-risk number represents two standard deviations below the mean ($100 – 2 × $30). This means that if the firm's earnings are normally distributed, the likelihood that pre-tax profit will be below $40 million is 2.275%, and the earnings-at-risk of $40 million translates into a 97.725% level of confidence that pre-tax profit will be acceptable

separate measures of credit risk capital, market risk capital and operational risk capital in the following ways:

- One of the 'credit capital models' (for example, Moody's-KMV Portfolio Manager or the RiskMetrics Group's Credit Manager) is used to determine credit risk capital.
- A value-at-risk model could be used to estimate market risk capital.
- There is not a generally accepted model for measuring operational risk capital. Some financial institutions use a 'process' approach to estimate operational risk capital, while others use a 'factor' approach and still others use an 'actuarial' approach.[5]

Some firms err on the side of conservatism by simply adding up credit risk capital, market risk capital and operational risk capital to estimate total economic capital. (By summing them, the firm is assuming that the risks are perfectly positively correlated. If they are less than perfectly positively correlated, total economic capital will be less than the sum.) However, some firms have begun devoting research to measuring the correlations between credit risk, market risk and operational risk.

□ **Imply from equity market data.** As Tierny & Smithson (2003) described in a recent article, the stock market can tell a firm how much capital it needs to support its assets. The volatility of the firm's stock price can be used in an option pricing model to simulate how different levels of equity capital will affect the volatility of the company's net assets, and thus the probability of financial distress.

Black & Scholes demonstrated that the value of equity is equivalent to a call option on the value of the firm's assets in which the exercise price is the face value of the debt.[6] If the firm's assets (that is, business units and activities) are held constant, and if we assume that only equity can be used to satisfy the firm's requirement for economic capital, we can use the option pricing model to determine how the market value of the shares would react to changes in the amount of equity. If a company's required economic capital is equal to the amount of equity it is currently holding, a change in equity will reduce shareholder value. (For example, if the firm were to increase equity in this case, the resulting increase in the value of the shares would be less than the increase in equity.) But if economic capital is greater than the firm's current amount of equity, an increase in equity is expected to increase shareholder value.

Task two: attribute economic capital to individual business units and activities

Attributing economic capital to the company's specific activities and business units provides the basis for value-maximising managerial decision-making. Senior management is confronted with a range of decisions about business unit X:

- How large a bonus should we pay the manager of business unit X?
- Is business unit X adding to shareholder value, or should it be sold?
- Can we increase the value of the firm by reallocating economic capital to or from business unit X?

To answer these questions, senior management must first know how much economic capital is needed to support business unit X.

Two measures of economic capital for individual business units have become well established:

- 'Stand-alone' economic capital – the amount of capital an individual business would require if it were an independent firm. As such, stand-alone economic capital does not reflect the beneficial effect of diversification on corporate risk. Consequently, the sum of stand-alone capital for the company's individual business units and activities will be greater than the total economic capital for the firm.
- 'Marginal' economic capital – the amount of capital that each business unit adds to the entire firm's capital requirement (or, conversely, the amount of capital that would be released if the business unit were sold). Sometimes referred to as the 'discrete marginal method', it is calculated by taking the economic capital required for the firm without the business unit and subtracting it from the economic capital required for the entire firm, including this business unit. The sum of marginal economic capital for all the firm's business units and activities will be smaller than total economic capital for the firm.

It is generally agreed that stand-alone capital is appropriate for measuring the performance of business unit managers, that is, the decision about how large a bonus to pay to the manager of business unit X.[7] It is also generally agreed that marginal economic capital is the appropriate measure for evaluating acquisitions or divestitures, that is, the decision about selling business unit X.

□ **'Marginal' versus 'diversified'.** While agreement exists regarding the appropriate economic capital for measuring the performance of the manager and for the entry/exit decision, there is much less agreement about the appropriate measure for pricing decisions and evaluating the performance of the business unit within the firm, that is, the decision about reallocating capital among the business units that make up the firm. Merton and Perold and other academics argue that marginal economic capital should be used.[8] However, practitioners appear to be using a measure of 'diversified economic capital'.

Diversified economic capital is intended to measure the amount of the firm's total capital that should be allocated to a particular business when viewed as part of a multi-business corporation. That is, the diversified capital measure is intended to allocate the diversification benefit among the business units and activities that make up the firm. Since the diversification benefits are allocated to the different business units and activities in the form of reduced economic capital, the sum of diversified economic capital for all the firm's business units and activities will be equal to total economic capital for the firm.

Proponents of the diversified economic capital approach point out that marginal capital always underallocates total firm capital and that, even if the marginal capital numbers were scaled up, the signals sent about profitability

To implement a risk-adjusted performance measure, the firm must be able to accomplish one difficult task and one extremely difficult task

*[5] These approaches to operational risk were covered in an earlier Class Notes column (*Risk *March 2000, pages 58–61)*

[6] To calculate the value of the company's shares as an option, one needs to come up with estimates of five variables: (1) the value of the company's net assets, (2) the volatility of net asset value – perhaps by 'unlevering' the firm's equity, (3) the time horizon, for example, an estimate of the average duration of the firm's net assets, (4) the risk-free interest rate with maturity corresponding to the time horizon, and (5) a 'trigger price' representing the value of the shareholder's equity at which the debt holders begin to demand repayment and possibly take steps to force the firm into bankruptcy

[7] This reflects the logic that decisions to diversify the corporate portfolio are generally not within the purview of operating managers

[8] Perold reminded me that this is subject to the constraint that the whole firm makes money

are potentially misleading.[9] Opponents of the diversified economic capital approach point out that, since "risk capital [is] not... additive across the businesses that comprise the firm, rules that fully allocate firm-wide risk capital across the constituent businesses are likely to be sub-optimal" (Perold, 2001).

□ **Measuring diversified capital.** The most widely used measure of diversified capital apportions risk on the basis of the covariance of each business unit with the entire organisation.[10] The diversification benefit a business gets credit for is a function of the correlations between the returns for that business unit and the firm – the lower the correlation, the greater the diversification benefit for the business unit (and the lower the amount of economic capital necessary to support it).

The variance (riskiness) of a firm comprised of N business units and the risk contribution (RC) of the ith business unit are[11]:

$$\sigma_P^2 \equiv \sum_{i=1}^{N}\sum_{j=1}^{N}\sigma_i\sigma_j\rho_{i,j}$$

$$RC_i \equiv \sigma_i\frac{\partial\sigma_P}{\partial\sigma_i}$$

The 'chain rule' (from differential calculus) implies that:

$$\frac{\partial\sigma_P^2}{\partial\sigma_i} = 2\sigma_P\frac{\partial\sigma_P}{\partial\sigma_i}$$

So, the risk contribution of the ith business unit simplifies to:

$$RC_i = \frac{\sigma_i}{2\sigma_P}\frac{\partial\sigma_P^2}{\partial\sigma_i} = \frac{\sigma_i}{2\sigma_P}\left(2\sigma_i + 2\sum_{j\neq i}\sigma_j\rho_{i,j}\right) = \sigma_i\times\frac{\sum_j\sigma_j\rho_{i,j}}{\sigma_P} = \rho_{i,P}\times\sigma_i$$

Note that the correlation of the business unit with the firm contains the standard deviations (σ_j) of all the business units and the correlations between all the pairs of business units ($\rho_{i,j}$):

$$\rho_{i,P} = \frac{1}{\sigma_P}\sum_{j=1}^{N}\sigma_j\rho_{i,j}$$

If we assume the underlying distribution is normal, total economic capital[12] is simply $k\sigma$, where $k = \Phi^{-1}(\alpha)$, $\Phi^{-1}(...)$ is the inverse cumulative standard normal distribution, and α is the confidence level (for example, 99%). In this case, diversified economic capital follows from the definition of diversified capital contribution (RC_i)[13]:

$$\left(EC_{DIVERSIFIED}\right)_i = \rho_{i,P}\times\left(EC_{STANDALONE}\right)$$

Opponents of the diversified economic capital approach note that this measure of diversified capital is based on two assumptions that cannot hold simultaneously, that is, it assumes that every dollar is marginal, while at the same time assuming that every other dollar is not marginal. Perold (2001) offers an example: if a firm held a $100 million bond position, this type of 'diversified economic capital' measure assumes the 100th million is as marginal as the first million – in each case assuming the other 99 million are in place – where, in fact, the first million of the position contributes much less to risk than the 100th million.

Recently, we have noted articles in which the tools of game theory are employed to identify 'fair' risk attributions (for example, Koyluoglu & Stoker, 2002, and Denault, 2001). Since the techniques of game theory have been applied to other problems involving the attribution of cost among a group (for example, telephone billing and airport landing fees), it makes sense to consider this technique for attributing economic capital to business units.

■ The Shapley method is a game theory technique that describes how coalitions could be formed so that a group of people (or, in our case, a group of business units) could benefit more as a group than if they worked separately. In this approach, a capital attribution becomes a cost. Each 'player' minimises their cost (capital attribution) and will leave the coalition if attributed a larger share of capital than their own stand-alone capital.

In most instances, the Shapley method yields results similar to the 'continuous marginal' approach. However, the problem with the Shapley method is that it involves a lengthy computation, so it may become impractical to calculate with even a modest number of business units.

■ The Aumann-Shapley (AS) value is a game theory allocation method that allows for 'fractional' players.[14] The advantage of the AS value is that it requires less computation, so is potentially more practical. Under fairly reasonable conditions, the AS value solution is the same as the continuous marginal approach. ■

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[9] Since the (discrete) marginal method is not additive, it cannot be directly applied for full risk decomposition. By 'scaling up' (that is, multiplying by a constant greater than one) all the marginal capital numbers, some business units may be allocated more risk capital than their stand-alone risk capital. For example, if a business unit were already highly correlated with the rest of the firm, its marginal capital would probably be very close to its stand-alone capital, and multiplying marginal capital by a constant greater than one may yield a risk allocation greater than the stand-alone capital, which would be unfair to that business line

[10] This methodology is sometimes called the 'continuous marginal' approach in the literature, because risk contribution derives from vanishingly small changes in the size of a constituent

[11] Note that the standard deviation of each business unit is expressed in units of currency (not returns) to reflect the size of the business unit within the firm. We can change to returns by substituting σ for $w\sigma'$, where σ' is the standard deviation of the returns and w is the weight of the business unit (or transaction) in the firm; the derivatives would then be taken with respect to the weight w, not σ'

[12] We are implicitly defining economic capital as a measure of the unexpected losses with respect to the mean of the loss distribution

[13] In this way, $EC_{DIVERSIFIED} = k \times RC$

[14] This assumption would be acceptable in our application, since it could be possible to sell part of a business unit