# Sticky-strike and sticky-delta[1] Greeks – casualties of the subprime crisis?

*Following its recent Foreign Exchange Options symposium, Murex presents a focus on implied dynamics from exotic option prices and the application to portfolio hedging*

**Does the flap of a butterfly's wings in Brazil set off a tornado in Texas?[2]**

The impact of the subprime crisis is not restricted to collateralised debt obligations and Bear Stearns, to name but two obvious examples. The UK's Northern Trust, with negligible direct subprime exposure, felt the draught from the subprime butterfly's wings. Similarly, an obvious casualty has been the US dollar (USD). Marginally less obvious was that material USD declines against the Japanese yen (JPY) would result in significant increases in at-the-money Volatility (ATMV) and perhaps, less obviously, to increasing skewness favouring USD puts.

Further removed and more esoteric effects occurred; increased skewness resulted in the inability to consistently solve smile curves from broker convention quotations. Many smile interpolation methods resulted in discontinuity noticeable as non-Gaussian gamma profiles, leaving participants without usable Greeks. Furthermore, when apparently satisfactory Greeks were generated, conventional delta hedging failed to adequately replicate (and therefore hedge) changes in option prices. Effectively the subprime butterfly resulted in USD/JPY option market-makers losing touch with their underlying market exposure.

The core crisis may be generally accepted as unexpected and by extension these 'butterfly effects' even more so. Surprisingly, historical analysis of market exotic prices before the crisis suggest the correlation between spot, volatility and skewness was accurately priced into USD/JPY exotics before the crisis. If dynamic hedging had taken into account the dynamics implicit in complex models used to price such options, the results would have been more satisfactory.

In response to market-makers' questions regarding these issues, Murex hosted a Foreign Exchange Options (FXO) symposium in London, bringing together practitioners – whether traders, quants or risk managers – Murex clients and non-clients alike. Several months of research on a range of topics was debated by a panel of head traders, quants and brokers with an audience representing many of the major names in the European FXO arena. Further panel discussions will occur at symposiums in New York and Singapore. This article focuses on implied dynamics from exotic option prices and the application to portfolio hedging.

*With acknowledgment to:*
[1] *Emanuel Derman,* Regimes of Volatility, *1999*
[2] *Edward Lorenz/Philip Merrilees,* Does the flap of a butterfly's wings in Brazil set off a tornado in Texas?, *1972*

**Skewness and correlation**

In the FXO market, skewness and leptokurtosis are represented by the 10 and 25 delta risk reversals ('RRs') (C-P) and butterflies ('Flies') (strangle – ATMV). Historical analysis shows negative correlation between volatility (ATMV), skewness (one-year 25 delta RR) and spot USD/JPY over a four-year period leading up to the subprime crisis. Correlation was stable for significant periods of time punctuated by several crises; indeed from September 2004 to April 2006 the correlation co-efficient is 93%, meaning 93% of the change in skewness was explained by changes in spot. The correlation was parabolic rather than linear, with changes in skewness increasing with lower levels of USD/JPY. Traders anecdotally associate the existence and magnitude of leptokurtosis with stochastic volatility and skewness with the correlation between spot and volatility. Murex's research across several currency pairs corroborated this, indicating that skewness was often associated with correlation in the direction of the skew, noting that, in the summer of 2005, when USD/EUR skewness changed sign, the ATMV/S correlation 'flipped' accordingly.

**Volatility dynamics implicit in complex models**

A general property of stochastic volatility and local volatility models is that they deduce volatility behaviours from the smile curves they are calibrated against. Further information about volatility is generally accepted to be implicit in first-generation exotic option prices. Once calibrated, complex models contain a volatility dynamic, whether simply regarding volatility or extending to skewness and/or leptokurtosis with respect to spot. This dynamic is typically not published in terms of observables such as ATMV, RRs and Flies, but parameters that depend on the specific diffusion of the model represent the sensitivity to volatility of volatility, initial variance, and so on.

One extension of functionality Murex integrated for the 2007 release of its stochastic/local volatility hybrid model, Tremor, is exposing the dATMVdS, dRRdS and dFlydS (25 and 10 delta) so that practitioners can intuitively understand the dynamic implicit in the model (or arguably implicit in market prices once the model has been calibrated against such) and compare that to actual market dynamics. This functionality was used to allow exotic prices before the subprime crisis to be used to generate implied volatility dynamics for comparison to observed dynamics during the decline.

1 Implied volatility dynamics versus USD/JPY spot ladder

Smile dynamics

Maturity 1Y | **USD/JPY** | Positive RR in favor of USD call | Absolute

| Spot (%) | Spot | ATMV | dATMV | 10 RR | dRR10 | 25 RR | dRR25 | 25 FLY | dFLY25 | 10 FLY | dFLY10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 5.00 | 129.5805 | 6.72 | 0.04 | -0.70 | 0.28 | -0.65 | 0.19 | 0.20 | -0.00 | 0.97 | -0.01 |
| 4.00 | 128.3464 | 6.67 | 0.04 | -0.98 | 0.28 | -0.84 | 0.19 | 0.21 | -0.00 | 0.98 | -0.01 |
| 3.00 | 127.1123 | 6.67 | 0.00 | -1.46 | 0.49 | -1.04 | 0.19 | 0.21 | -0.01 | 1.02 | -0.03 |
| 2.00 | 125.8782 | 6.72 | -0.04 | -1.97 | 0.51 | -1.24 | 0.20 | 0.22 | -0.01 | 1.07 | -0.06 |
| 1.00 | 124.6441 | 6.80 | -0.09 | -2.51 | 0.54 | -1.46 | 0.22 | 0.23 | -0.01 | 1.15 | -0.08 |
| **0.00** | **123.4100** | **6.92** | **-0.12** | **-3.10** | **0.59** | **-1.69** | **0.23** | **0.27** | **-0.04** | **1.25** | **-0.10** |
| -4.18 | 118.2502 | 8.01 | -0.26 | -4.17 | 0.26 | -2.97 | 0.31 | 0.38 | -0.03 | 2.07 | -0.20 |
| -5.52 | 116.6002 | 8.55 | -0.41 | -8.02 | 2.88 | -3.53 | 0.42 | 0.45 | -0.05 | 2.51 | -0.33 |
| -6.91 | 114.8824 | 9.23 | -0.48 | -9.96 | 1.40 | -4.22 | 0.50 | 0.54 | -0.06 | 3.09 | -0.42 |
| -7.32 | 114.3764 | 9.45 | -0.53 | -10.60 | 1.56 | -4.45 | 0.55 | 0.57 | -0.07 | 3.29 | -0.48 |
| -7.71 | 113.8951 | 9.67 | -0.57 | -11.26 | 1.68 | -4.68 | 0.59 | 0.60 | -0.08 | 3.49 | -0.52 |

Dynamic definition | Dynamic graph | Save | Cancel

### Implied versus actual volatility dynamics during the subprime USD/JPY decline

Tremor was calibrated shortly before the major declines (July 9, 2007) against baskets of USD/JPY one touches (digitals or binaries), with spot at 123.41, resulting in a negative implied correlation between ATMV, RR and spot. The spot ladder depicted in figure 1 shows the effects of these 'market-implied correlations' against five 1% steps on the upside, and against actual spot prices observed during the period Monday August 13 to Monday August 20 on the downside, when USD/JPY declined by approximately 7% versus the 123.41 rate used during calibration on July 9. Note that the dVdS and dRRdS accelerate as USD/JPY declines, which is consistent with the parabola observed in historic analysis:

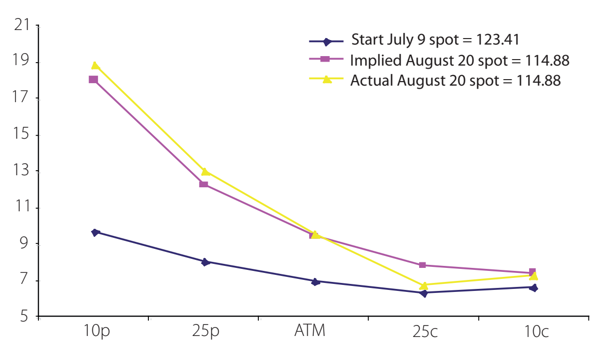
- On July 9 with USD/JPY 123.41; ATMV = 6.92%; RR10 = 3.1% and RR25 = 1.69%
- On August 20 with USD/JPY 114.88; ATMV = 9.6%; RR10 = 11.5% and RR25 = 6.25%
- This compares to volatilities implied from the exotic options' volatility dynamic given a spot level on USD/JPY of 114.88 of ATMV = 9.23%; RR10 = 9.96% and RR25 = 4.22%.

Both the direction and magnitude of changes in volatility and skewness were materially implicit in market prices of July 9, as depicted in figure 2. Of course, given a crisis, differences are unsurprising, in this case the minor nature of them is more surprising.

### Greeks consistent with smile dynamics

Notwithstanding that many practitioners use complex models to value exotic options, the majority revert to 'sticky-strike' or 'sticky-delta' Greeks when hedging rather than embracing Greeks stemming from such models incorporating volatility dynamics. Effectively, books are hedged using Greeks that imply no change in volatility or skewness for a change in spot, although this is anecdotally anticipated and historically demonstrated, as well as implicit in the volatility surface and market exotic prices. Analysis presented highlighted the significant differences in delta between Black-Scholes/no smile, smiled volatility/sticky strike (the lingua franca of the FXO market as the broker or exchange delta), smiled volatility/sticky-delta and smile-dynamic-consistent deltas. For example, given August 13 market data, a one-year USD/JPY 10 delta put (a put with a -10% broker or exchange sticky-strike delta) has a -3% delta under pure Black-Scholes without smile, -6% under smiled/sticky delta and -20% when the smile dynamic is incorporated. Effectively, the smile dynamic delta assumes a decline in USD/JPY will be accompanied by an increase in ATMV and the 10 delta Fly and RR, all increasing the volatility and hence premium of the option more significantly that a change in spot in isolation, thereby requiring the option seller to establish a larger short position.

2 Implied volatility levels versus actual volatility levels




### Conclusion

It is fraught with danger to draw conclusions from any limited scope analysis, but it seems that, between the quants' complex models and traders' market assessment, FXOs are priced efficiently using volatility dynamics that closely reflect actual market dynamics. While no one will suggest that the USD/JPY crisis was predictable, it is arguable that the volatility dynamics associated with spot movement were already well understood, rather than an unexpected butterfly effect resulting from subprime chaos. Practitioners short of volatility and/or downside in USD/JPY would have fared much better through this crisis using Greeks that embraced dynamics consistent with their complex models rather than applying conventional sticky-strike or sticky-delta regimes. In many cases, practitioners implicitly acknowledge this and establish delta positions against vega risk to heuristically hedge the change in volatility associated with a change in spot. Problematically, this results in banks' reported risk differing from the traders' assessment (which is perhaps more consistent with the banks' approved exotic models). Is it time for a more broadly spread evolution of the volatility regimes embedded in market risk assumptions?